Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Coupled Radiation with Turbulent Convection in Electric Arcs

R. Pyare* and M. M. Abu-Romia†
Polytechnic Institute of Brooklyn, Brooklyn, N.Y.

Introduction

The satisfactory performance of plasma arc devices has been known to be limited by Joule heating, as well as thermal conduction, turbulent convection and radiation. The effect of turbulence has not been included in previous investigations, and this may restrict the validity of the reported results to arcs operating with small flow rates. The present paper accounts for the coupling of turbulence with radiation in arcs, with results shown to indicate the relative role of each at various values of Reynolds number and operating current.

Analysis

Consider a steady flow in a cooled, constant wall-temperature tube whose internal surface is black. The body forces and viscous dissipation are negligible. The plasma is assumed to be in a state of local thermodynamic equilibrium. The axial field intensity is constant, and diffusion of ions and electrons is ambipolar.

The governing equations in the fully developed stage are

$$F = -(\mu + \rho\varepsilon_M)\, du/dr \tag{1}$$

$$(1/r)\, d/dr\,(rq) = \delta E^2 \tag{2}$$

$$q = q_R - (\lambda + \rho c_p \varepsilon_H)\, dT/dr \tag{3}$$

$$q_c = -\lambda\, dT/dr \tag{4}$$

Where c_p is the specific heat at constant pressure, E is the field intensity, F is the shear stress, q is the total heat flux, q_c is the conduction heat flux, q_R is the total radiation heat flux, r is the radial coordinate, T is the absolute temperature, u is the velocity, ε_M and ε_H are the turbulent diffusivities for momentum and heat, respectively, λ is the thermal conductivity, δ is the electrical conductivity, μ is the molecular viscosity, and ρ is the density. The boundary conditions are

i) $u = 0,\ r = r_0$ ii) $q = 0,\ r = 0$ iii) $T = T_0,\ r = r_0$

the suffix 0 used here and later on refers to the quantities evaluated at the wall. The assumption that turbulence has direct effect only on viscosity and thermal conductivity has been justified by Harder and Cann.[1]

The general expression for the radiation heat flux taking into account the spectral dependence of the absorption coefficient is given by

$$q_R(r) = \frac{4}{\pi}\int_0^{\pi/2} \cos(\gamma)\left\{\int_{r\sin(\gamma)}^{r_0} \frac{de}{dr'}\Big(-\varepsilon[T(r'),\ (r+r'-2r\sin(\gamma))/\cos(\gamma)] + \varepsilon[T(r'),\ |r-r'|/\cos(\gamma)]\Big)\, dr'\right\} d\gamma \tag{5}$$

where

$$\varepsilon[T(r'), X] = \int_0^\infty \frac{de_\omega(r')}{de(r')}\left(1 - D_3[K_\omega(r')X]\right) d\omega$$

$$D_n(X) = \int_0^{\pi/2} \cos^{n-1}(\alpha)\exp\left(-X/\cos(\alpha)\right) d\alpha \tag{6}$$

Here e is the black body emissive power ($=\sigma T^4$, σ is called Stefan-Boltzmann's constant), e_ω is the spectral emissive power, K_ω is the monochromatic absorption coefficient, and ε is called the modified emissivity. The details of the derivation of Eq. (5) are given by Pyare and Abu-Romia.[2]

In accordance with Ref. 3, eddy diffusivity is taken as

$$\varepsilon_M = k_1{}^2 uy\left[1 - \exp\left(-\frac{k_1{}^2 uy}{\mu/\rho}\right)\right] \quad \text{for} \quad y^+ \leqq 26 \tag{7}$$

where $k_1 = 0.124$. For $y^+ > 26$

$$\varepsilon_M = k_2(F_0/\rho)^{1/2} y(1 - y/r_0) - \mu/\rho \tag{8}$$

assuming linear shear stress distribution, and

$$\varepsilon_M = k_2(K_0/\rho)^{1/2} y - \mu/\rho \tag{9}$$

assuming constant shear stress distribution. Here $k_2 = 0.36$, $y = r_0 - r$ and $y^+ = (F_0/\rho_0)^{1/2}/(\mu_0/\rho_0)$. The equality of eddy diffusivities is postulated in the analysis, i.e., $\varepsilon_M = \varepsilon_H$.

In the computations, the transport and thermodynamic properties were taken from Yos[4] and the monochromatic absorption coefficient from Churchill et al.[5] The results were obtained by assuming a polynomial for the fourth power of the temperature,

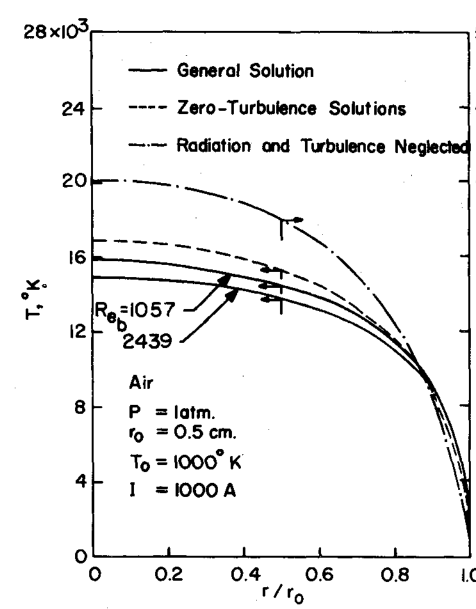

Fig. 1 Effect of turbulence on temperature profiles.

Presented as Paper 72-685 at the 5th Fluid and Plasmas Dynamics Conference, Boston, Mass., June 26–28, 1972; submitted June 30, 1972; revision received March 1, 1973. This is taken from the dissertation submitted by R. Pyare to the faculty of the Polytechnic Institute of Brooklyn in partial fulfillment of the requirements for the degree, Doctor of Philosophy (Mechanical Engineering), 1971.

Index categories: Radiatively Coupled Flows and Heat Transfer; Plasma Dynamics and MHD.

* Teaching Fellow; presently Research Engineer, Baker Brush Company, New York.

† Associate Professor, Department of Mechanical Engineering. Member AIAA.

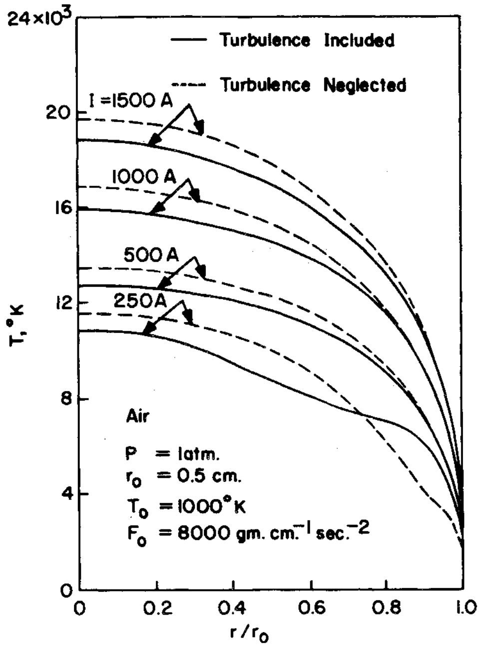

Fig. 2 Temperature profiles for various values of current.

with the coefficients being obtained by an iterative scheme. Details of the numerical results are given in Ref. 6. The current I, indicated in Figs. 1–3, is related to the voltage by the over-all energy balance

$$2\pi r_0 q_0 = EI \tag{10}$$

Values of Reynolds number Re_b, mass flow rate m, and the mean bulk enthalpy are calculated from

$$Re_b = 2m/(\pi r_0 \mu_b) \tag{11}$$

$$m = 2\pi \int_0^{r_0} \rho u r \, dr \tag{12}$$

$$H_b = \left(2\pi \int_0^{r_0} H \rho u r \, dr\right) \Big/ m \tag{14}$$

where μ_b is the viscosity evaluated at the bulk temperature.

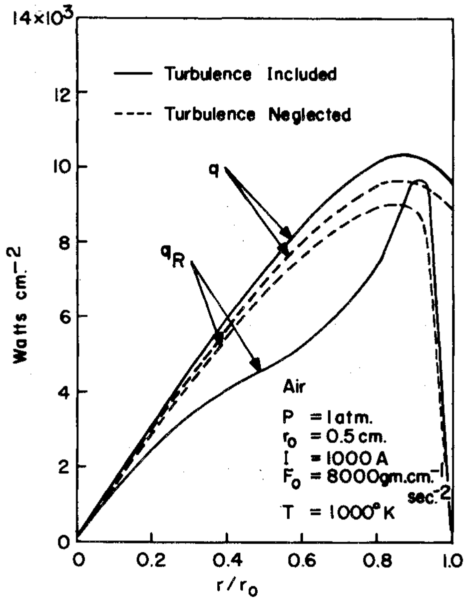

Fig. 3 Effect of turbulence on heat flux profiles.

Results

As indicated in Fig. 1, the effect of turbulence, even for a relatively high Reynolds number, is small compared to the effect of radiation which is the dominant mechanism at high currents. At lower operating currents, i.e., low plasma temperatures, the effect of turbulence is more significant while radiation has a diminishing role. As can be noted in Fig. 2, the temperature profile with turbulence is flatter than that without turbulence. Fig. 3 shows that turbulence has significant effect on radiation flux distribution, and as expected, the total heat flux at the wall is increased by the presence of turbulence.

References

[1] Harder, R. L. and Cann, G. L., "Correlation of High-Pressure Arc Heater Results," *AIAA Journal*, Vol. 8, No. 12, Dec. 1970, pp. 2220–2225.

[2] Pyare, R. and Abu-Romia, M. M., "Plasma Radiation Effects in Tube Arc Heating," ASME Paper 71-HT-18, Tulsa, Okla., 1971.

[3] McEligot, D. M., Smith, S. W., and Bankston, C. A., "Quasi-Developed Turbulent Pipe Flow with Heat Transfer," *Journal of Heat Transfer*, Vol. 92, No. 4, Nov. 1970, pp. 641–650.

[4] Yos, J. M., "Transport Properties of Nitrogen, Hydrogen, Oxygen and Air to 30,000°K," RAD-TM-63-7, March 1963, AVCO Corp., Wilmington, Mass.

[5] Churchill, D. R., Armstrong, B. H., Johnson, R. R., and Miller, K. G., "Absorption Coefficient of Heated Air, a Tabulation to 24,000°K," *Journal of Quantitative Spectroscopy and Radiative Transfer*, Vol. 6, No. 4, April 1966, pp. 371–442.

[6] Pyare, R., "Theoretical Investigation of Energy Transfer in Plasma Arc Devices," Ph.D. dissertation, June 1971, Dept. of Mechanical Engineering, Polytechnic Inst. of Brooklyn, Brooklyn, N.Y.

Dynamically-Induced Large Deformations of Multilayer, Variable Thickness Shells

JOHN W. LEECH*
Boston University, Boston, Mass.
AND
EMMETT A. WITMER†
Massachusetts Institute of Technology, Cambridge, Mass.
AND
LUIGI MORINO‡
Boston University, Boston, Mass.

IN recent years the analysis and prediction of large transient and permanent deformations of structures have been of increasing interest in connection with explosive and/or magnetic forming of simple and complex structural shapes. Also various aerospace, water, and land vehicles as well as stationary structures may be subject to intense transient loading caused by

Received November 6, 1972. This research was sponsored at MIT by Ballistic Research Laboratories, USARDC, Aberdeen Proving Grounds, Md. under Contract DAADO5-68-C-0314, and was supervised by N. J. Huffington Jr. and J. Santiago of BRL. The authors with to acknowledge the contributions of this work by a former colleague, S. Atluri, now of the University of Washington.

Index categories: Aircraft Structural Design (Including Loads); Structural Dynamic Analysis; Thermal Stresses.

* Associate Professor of Aerospace Engineering. Associate Fellow AIAA.

† Professor of Aeronautics and Astronautics. Member AIAA.

‡ Associate Professor of Aerospace Engineering. Member AIAA.